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Electromagnetic

Lecture (6)

مركز الفجر للخدمات العلمية ٠١٠٠٩٢٦٤٠٠٦

Chapter (3)

"Energy and Potential"

2nd year
communications

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Lecture (6) :

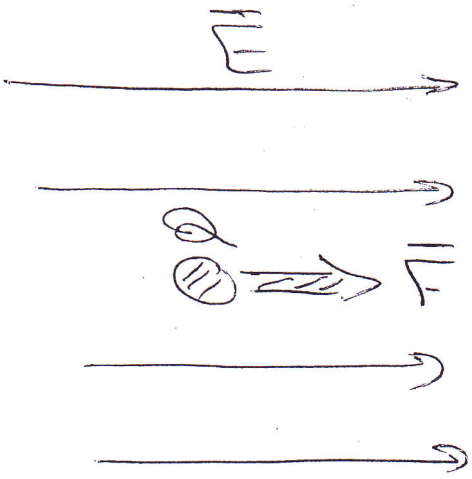
Chapter (3)

Energy and potential

[1] Work done in moving a point charge in an electric field :

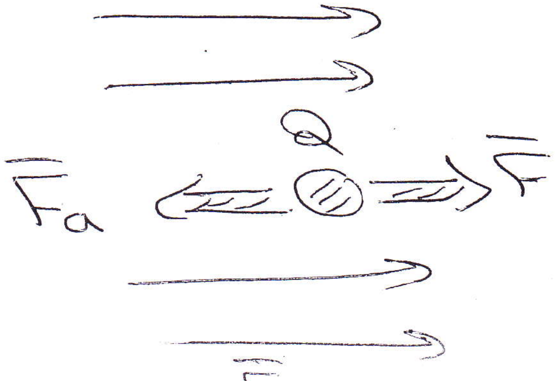
• عند وضع شحنة Q في مجال كهربي $\bar{E}$ فإنها تتأثر بقوة $\bar{F}$ ناشئة عن هذا المجال.

$$\bar{F} = Q\bar{E}$$



• ولكي تتزن هذه الشحنة (أي لا تتحرك) لابد من بذل قوة F_a مساوية لـ $\bar{F}$ وفي عكس إتجاهها.

$$\bar{F}_a = -\bar{F}$$
$$= -Q\bar{E}$$



- إذا أردنا تحريك هذه الشحنة مسافة قدرها $d\bar{L}$ فإننا نحتاج لبذل شغل مقداره dW

- Work is defined as a force acting over a distance.

$$dW = \bar{F}_a \cdot d\bar{L} = -Q\,\bar{E}\cdot d\bar{L}$$

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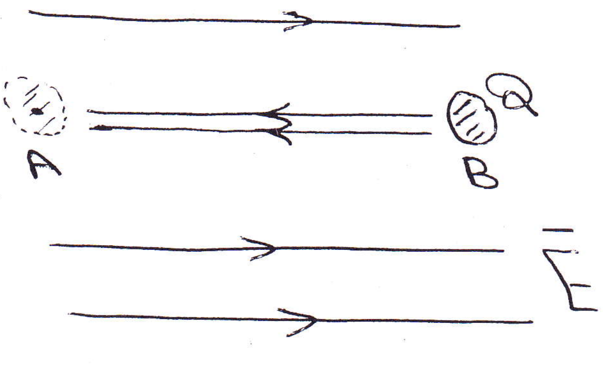
$dw = F_a \cdot dL$

$$\boxed{\therefore W_{AB} = -Q\int_{B}^{A} \bar{E}\cdot d\bar{L}}$$



W_{AB}: الشغل اللازم لنقل شحنة Q خلال مجال كهربي $\bar{E}$ من النقطة B إلى A.

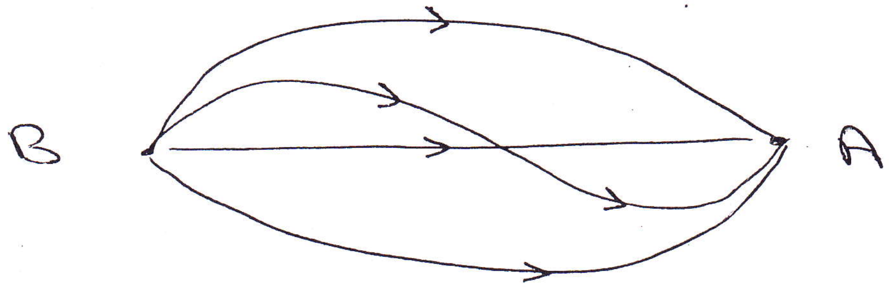
$$\rightarrow d\bar{L} = h_1\,du_1\,\hat{u}_1 + h_2\,du_2\,\hat{u}_2 + h_3\,du_3\,\hat{u}_3$$

$$\therefore d\bar{L} = \begin{cases} dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z} & \text{Cart.} \\ d\rho\,\hat{\rho} + \rho\,d\phi\,\hat{\phi} + dz\,\hat{z} & \text{Cyl.} \\ dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi} & \text{Sph.} \end{cases}$$

III) Conservative property of the electrostati field:

- The work done in moving a point charge from point B to A in a static $\bar{E}$ is independent on the path taken.

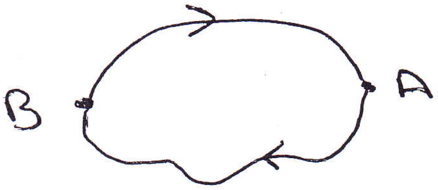
لايعتمد مقدار الشغل على المسار من B إلى A



"جميع هذه المسارات لها نفس مقدار الشغل"

ينتج من ذلك أن الشغل المبذول لتحريك شحنة على مسار مغلق = صفر

$$\boxed{W = \oint Q\bar{E} \cdot d\bar{L} = 0}$$

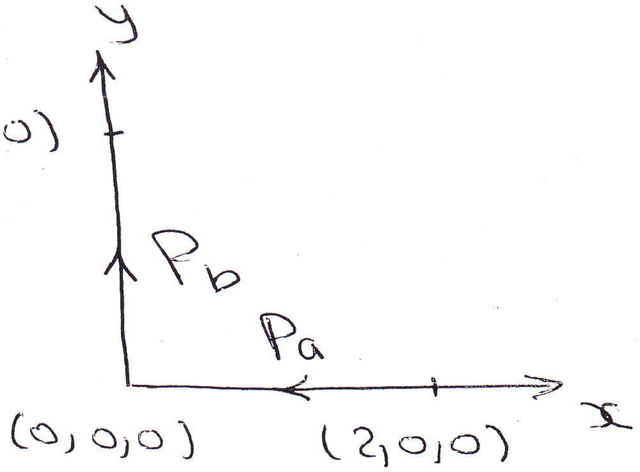


B: intial and Final Point.

Sheet (4)

[1] $Q = 2C$, $\bar{E} = 2x\hat{x} - 4y\hat{y}$ V/m

ⓐ Path 1 :



$dW = -Q\bar{E} \cdot d\bar{L}$

$d\bar{L} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$$(2,0,0) \xrightarrow[\substack{y=0 \\ dy=0 \\ x:\,2 \to 0}]{P_a} (0,0,0) \xrightarrow[\substack{x=0 \\ dx=0 \\ y:\,0 \to 2}]{P_b} (0,2,0)$$

P_a :

$$dW_a = -2\left[2x\hat{x} - 4\cancelto{0}{y}\,\hat{y}\right] \cdot \left[dx\,\hat{x} + \cancelto{0}{dy}\,\hat{y} + dz\,\hat{z}\right]$$

$$= -4x\,dx$$

$$\therefore W_a = \int_2^0 -4x\,dx = 8 \text{ J}$$

P_b :

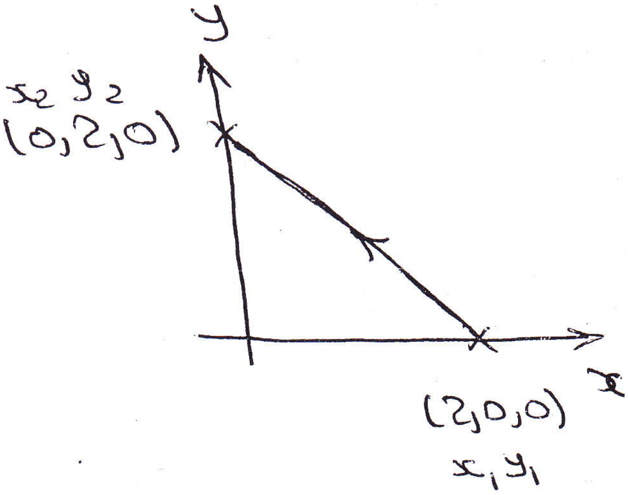
$$dW_b = -2\left[2\cancelto{0}{x}\,\hat{x} - 4y\,\hat{y}\right] \cdot \left[\cancelto{0}{dx}\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right]$$

$$= 8y\,dy$$

$$\therefore W_b = \int_0^2 8y\,dy = 16\text{ J}$$

$$\therefore W_1 = W_a + W_b = 8+16 = 24\text{ J}$$

ⓑ Path 2 :



eqn :

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\therefore \frac{y-0}{x-2} = \frac{2-0}{0-2} = -1$$

$$\therefore y+x=2 \Rightarrow y=2-x$$

$$\therefore dW_2 = -2[2x\hat{x} - 4y\hat{y}]\cdot[dx\hat{x}+dy\hat{y}+dz\hat{z}]$$

$$= -2[2x\,dx - 4y\,dy]$$

$$\therefore W_2 = -2\left[\int_2^0 2x\,dx - \int_0^2 4y\,dy\right]$$

$$= -2[-4-8] = 24\text{ J}$$

$\boxed{2}$ The electric potential :

$dw = QEdl$

$VAw = -Edl$

⇛ The potential differenCe :

فرق الجهد بين نقطتين : هو مقدار الشغل المبذول لنقل وحدة الشحنات الموجبة من إحدى النقطتين الى الأخرى

$$V_{AB} = \frac{W_{AB}}{Q} = -\int_{B}^{A} \bar{E} \cdot d\bar{L}$$

$-\phi$

$$= V_A - V_B$$

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⇛ The absolute potential :

الجهد المطلق لنقطة ما : هو مقدار الشغل المبذول لنقل وحدة الشحنات الموجبة من ال ∞ الى النقطة

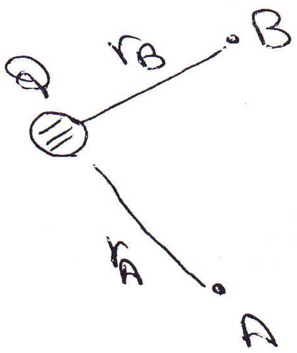
$$V_A = -\int_{\infty}^{A} \bar{E} \cdot d\bar{L}$$

"وكأن ال refrenCe هنا هو ال ∞"

2 $Q = 500\,pC$, $r_A = 5\,m$, and $r_B = 15\,m$

Sheet (4)

$$\bar{E}_Q = \frac{Q}{4\pi\epsilon r^2}\,\hat{r}$$



$$d\bar{L} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\therefore V_{AB} = -\int_{r_B}^{r_A} \bar{E}\cdot d\bar{L}$$

$$= -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon r^2}\,dr = \frac{-Q}{4\pi\epsilon}\cdot\frac{r^{-1}}{-1}\Bigg|_{r_B}^{r_A}$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$= V_A - V_B$$

$$V_A = \frac{Q}{4\pi\epsilon r_A} = 0.8987\ V$$

$$V_B = \frac{Q}{4\pi\epsilon r_B} = 0.2996\ V$$

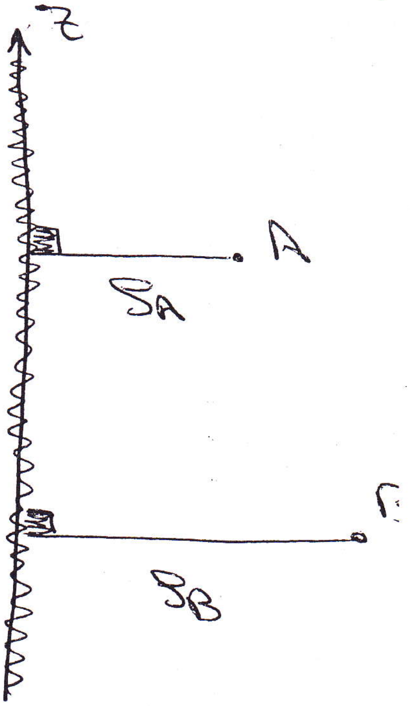
$$\therefore V_{AB} = 0.5992\ V$$

$\boxed{3}$ $\rho_L = 0.5\ nC/m$ (z-axis)

A(2, π/2, 0) and B(4, π, 5)

$\Rightarrow \bar{E}_l = \dfrac{\rho_L}{2\pi\epsilon\rho}\hat{\rho}$ (infinite + ρ_L = const)

$d\bar{L} = d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{z}$ (Cyl.)



$$\therefore V_{AB} = -\int_{\rho_B}^{\rho_A} \bar{E}\cdot d\bar{L}$$

$$\therefore V_{AB} = -\int_{\rho_B}^{\rho_A} \frac{\rho_L}{2\pi\epsilon\rho}\, d\rho$$

$$= -\frac{\rho_L}{2\pi\epsilon}\cdot \ln\rho \Big|_{\rho_B}^{\rho_A}$$

$$= -\frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{\rho_A}{\rho_B}\right)$$

$\rho_A = 2$ & $\rho_B = 4$

$\therefore V_{AB} = 6.23\ V$

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[3] The Equipotential Surface:

السطح متساوي الجهد : هو سطح أي نقطة عليه تكون لها نفس الجهد بمعنى أننا لا نحتاج بذل شغل لنقل شحنة على هذا السطح من نقطة لأخرى.

⇒ Equipotential sur. eqn:

$$V = \text{Const.}$$

⇒ Special Cases:

Charge	equipotential sur.
Point sphere	Spheres
line Cyl.	Cyl.

[4] The potential gradiant:

$$\bar{E} = -\nabla V$$

5 Energy density in the electrostatic field:

(a) point charge:

$$W_E = \frac{1}{2} \sum_{i=1}^{N} Q_i V_i \Rightarrow \text{Stored Energy (J)}$$

N: no. of charges.

(b) Distributed charge:

Find $\bar{E}$ using Gauss or any other method, then

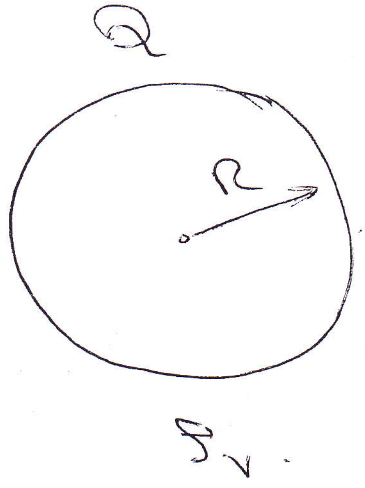
$$W_E = \frac{\epsilon}{2} \int_v |E|^2 \, dv \Rightarrow \text{Stored Energy (J)}$$

$\Rightarrow$ Stored Energy Density $= \frac{\epsilon}{2} |E|^2$

$\Rightarrow$ (J/m^3)

Sheet (4)

|4|



(a) Distributed Charge:

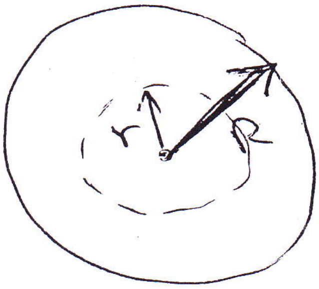
Find $\bar{E}$ using Gauss law:

$$\rho_v = \frac{\text{Charge}}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

Charge: sph.

Gaussian sur: sph with radius r.

· $r < R$:



$$Q_{enc} = \rho_v \cdot \text{vol}$$

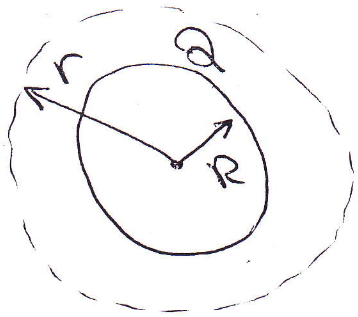
$$= \frac{3Q}{4\pi R^3} \cdot \frac{4}{3}\pi r^3$$

$$= \frac{Qr^3}{R^3}$$

$$\bar{D} \cdot \bar{S} = 4\pi r^2 D_r \rightarrow 4\pi r^2 D_r = \frac{Qr^3}{R^3}$$

$$\therefore \bar{D} = \frac{Qr}{4\pi R^3}\hat{r} \qquad \boxed{\bar{E} = \frac{Qr}{4\pi \epsilon R^3}\hat{r}}$$

- $r > R$:



$$Q_{enc} = Q$$

$$\therefore Q = 4\pi r^2 D_r$$

$$\bar{D} = \frac{Q}{4\pi r^2}\hat{r}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2}\hat{r}$$

$$\therefore \bar{E} = \begin{cases} \dfrac{Qr}{4\pi\epsilon R^3}\hat{r} & r < R \\ \dfrac{Q}{4\pi\epsilon r^2}\hat{r} & r > R \end{cases}$$

$\Rightarrow$ Stored energy : $W_E = \frac{\epsilon}{2}\int_v |E|^2 dv$

$$\therefore W_E = \frac{\epsilon}{2}\left[\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \left|\frac{Qr}{4\pi\epsilon R^3}\right|^2 r^2 \sin\theta \, dr\, d\theta\, d\phi \right.$$

$$\left. + \int_0^{2\pi}\int_0^{\pi}\int_R^{\infty} \left|\frac{Q}{4\pi\epsilon r^2}\right|^2 r^2 \sin\theta \, dr\, d\theta\, d\phi\right]$$

$$W_E = \frac{\epsilon}{2}\left[\frac{2\pi Q^2}{16\pi^2\epsilon^2R^6}\int_0^\pi\int_0^R r^4 \sin\theta \, dr\, d\theta\right.$$

$$\left.+ \frac{2\pi Q^2}{16\pi^2\epsilon^2}\int_0^\pi\int_R^\infty \frac{\sin\theta}{r^2}\, dr\, d\theta\right]$$

$$= \frac{\epsilon}{2}\cdot\frac{Q^2}{8\pi\epsilon^2}\left[\frac{1}{R^6}\cdot\frac{r^5}{5}\Big|_0^R\cdot\cos\theta\Big|_\pi^0\right.$$

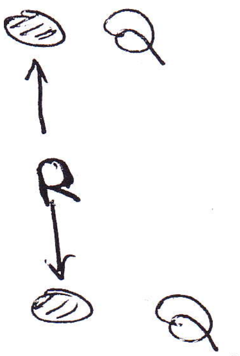
$$\left.+\frac{1}{r}\Big|_\infty^R\cdot\cos\theta\Big|_\pi^0\right]$$

$$=\frac{Q^2}{16\pi\epsilon}\left[\frac{1}{R^6}\cdot\frac{2}{5}R^5+\frac{1}{R}*2\right]$$

$$=\frac{Q^2}{16\pi\epsilon}\left[\frac{2}{5R}+\frac{2}{R}\right]$$

$$\therefore W_E = \boxed{\frac{3Q^2}{20\pi\epsilon R}}$$

(b) $W_E = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i$ $\boxed{N=2}$



$$\therefore W_E = \frac{1}{2}\left[Q_1V_1 + Q_2V_2\right]$$

but $V_1 = V_2 = \dfrac{Q}{4\pi\epsilon R}$

$$\therefore W_E = \frac{1}{2}\left[2 \cdot \frac{Q^2}{4\pi\epsilon R}\right] = \frac{Q^2}{4\pi\epsilon R}$$

$$\frac{W_E\big|_{sph}}{W_E\big|_{Q}} = \frac{3Q^2}{20\pi\epsilon R} \cdot \frac{4\pi\epsilon R}{Q^2} = 0.6$$

$$W_E\big|_{sph} = 60\% \text{ of } W_E\big|_{Q}$$

$\boxed{6}$ $\quad V = -\dfrac{60\,\phi}{\pi}\ \text{v}$

$$0.1 \le \rho \le 0.6$$
$$0 \le \phi \le \pi/6$$
$$0 \le z \le 1$$

$$\bar{E} = -\nabla V = -\left[\frac{1}{h_1}\frac{\partial V}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial V}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial V}{\partial u_3}\hat{u}_3\right]$$

$$= -\left[\cancelto{0}{\frac{\partial V}{\partial \rho}}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\phi} + \cancelto{0}{\frac{\partial V}{\partial z}}\hat{z}\right]$$

$$= -\frac{1}{\rho} \cdot \frac{-60}{\pi}\hat{\phi}$$

$$\therefore \boxed{\bar{E} = \frac{60}{\pi\rho}\hat{\phi}}$$

$$W_E = \frac{\epsilon}{2}\int_V |E|^2 \, dv$$

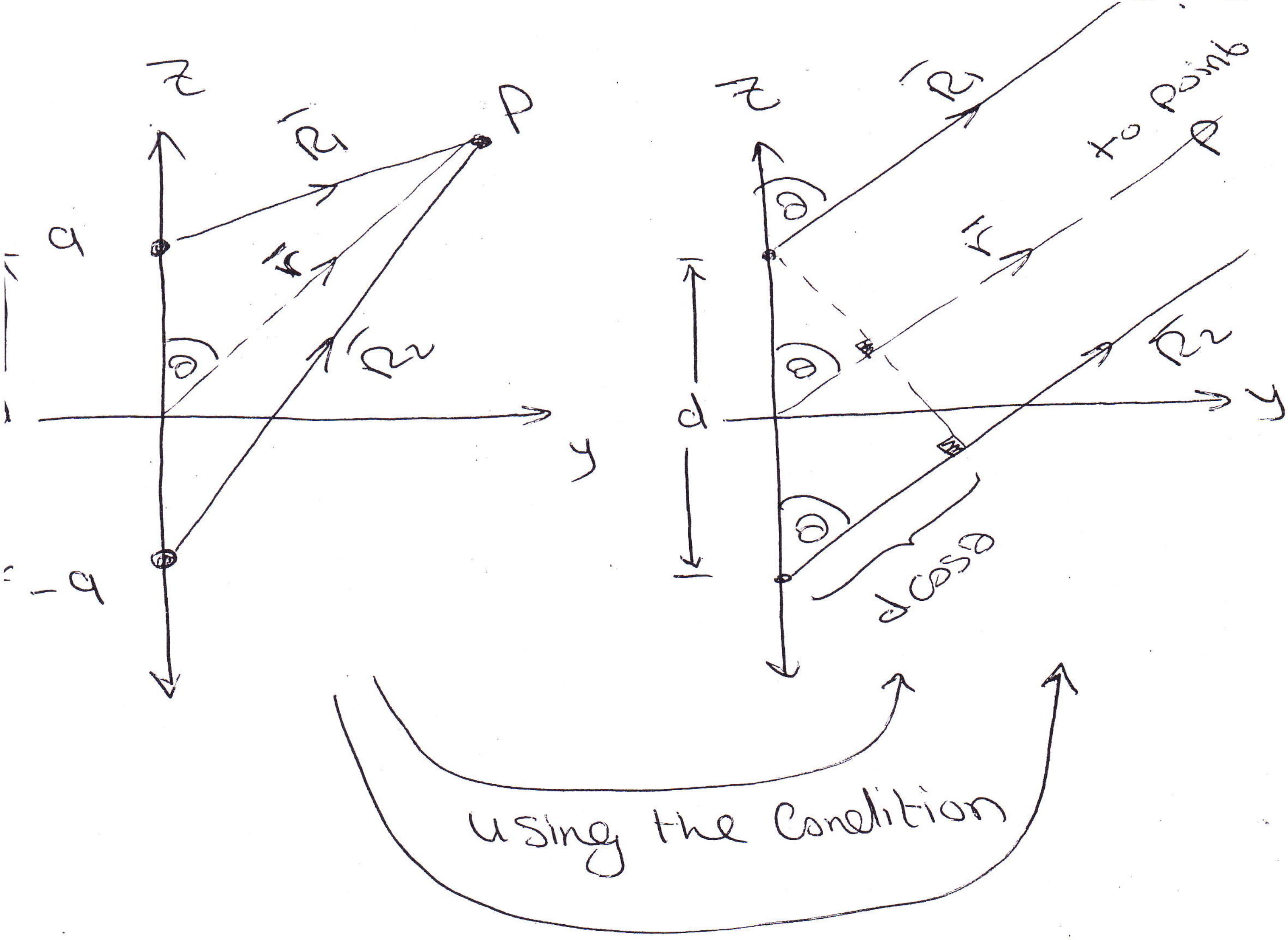
$$W_E = \frac{\epsilon}{2}\int_0^1 \int_0^{\pi/6} \int_{0.1}^{0.6} \left(\frac{60}{\pi \rho}\right)^2 \rho \, d\rho \, d\phi \, dz$$

$$= \left(\frac{60}{\pi}\right)^2 \cdot \ln\rho \Big|_{0.1}^{0.6} \cdot \phi \Big|_0^{\pi/6} \cdot \frac{\epsilon}{2}$$

$$= \boxed{1.515 \text{ nJ}}$$

|6| The Dipole :

• هو عبارة عن شحنتين متساويتين في المقدار و مختلفتين في الإشارة تفصل بينهما مسافة d بشرط أن تكون هذه المسافة أقل بكثير من المسافة (r) بين هذا الـ dipole والنقطة التي نحسب عندها المجال.



Sheet (4) : prob (7)

Point charge: $V_Q = \dfrac{Q}{4\pi\epsilon |R|}$

$$\therefore V_P = \frac{q}{4\pi\epsilon R_1} + \frac{-q}{4\pi\epsilon R_2}$$

$$= \frac{q}{4\pi\epsilon} \cdot \left[\frac{R_2 - R_1}{R_1 R_2} \right] \longrightarrow (1)$$

From the fig:

$$r = R_1 + \frac{d}{2}\cos\theta \longrightarrow (2)$$

$$R_2 = r + \frac{d}{2}\cos\theta \longrightarrow (3)$$

$$R_2 = R_1 + d\cos\theta \longrightarrow (4)$$

From (2, 3, 4) in (1):

$$V_P = \frac{q}{4\pi\epsilon} \left[\frac{d\cos\theta}{\left(r - \frac{d}{2}\cos\theta\right)\left(r + \frac{d}{2}\cos\theta\right)} \right]$$

$$= \frac{q}{4\pi\epsilon} \left[\frac{d\cos\theta}{r^2 - \underbrace{\frac{d^2}{4}\cos^2\theta}_{\text{neglect}}} \right]$$

Condition: $r >> d$

$$\therefore \boxed{V_p = \frac{qd\cos\theta}{4\pi\epsilon r^2}}$$

$$\because \bar{E} = -\nabla V$$

$$= -\left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}\right]$$

$$\rightarrow \frac{\partial V}{\partial r} = \frac{qd\cos\theta}{4\pi\epsilon}\cdot\frac{-2}{r^3} = -\frac{qd\cos\theta}{2\pi\epsilon r^3}$$

$$\rightarrow \frac{\partial V}{\partial \theta} = -\frac{qd\sin\theta}{4\pi\epsilon r^2}$$

$$\rightarrow \frac{\partial V}{\partial \phi} = 0$$

$$\therefore \bar{E} = \frac{qd\cos\theta}{2\pi\epsilon r^3}\hat{r} + \frac{qd\sin\theta}{4\pi\epsilon r^3}\hat{\theta}$$

$$\boxed{\bar{E}_p = \frac{qd}{4\pi\epsilon r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]}$$

$\Rightarrow$ Stream line eq^n:

$$\frac{dr}{E_r} = \frac{r\,d\theta}{E_\theta} = \frac{r\sin\theta\,d\phi}{E_\phi} \qquad \text{(Sph)}$$

$$\therefore \frac{dr}{2\cos\theta} = \frac{r\,d\theta}{\sin\theta}$$

$$\int \frac{dr}{r} = 2\int \frac{\cos\theta}{\sin\theta}\,d\theta$$

البسط تفاضل المقام

$$\therefore \ln r = 2\ln(\sin\theta) + \underbrace{\ln C}_{\text{ثابت}}$$

$$\therefore \boxed{r = C\sin^2\theta}$$

⇛ Dipole moment: $\bar{P}$

$\bar{P}$ is a vector

mag. $= qd$

direction from $-q$ to q $= \hat{z}$

$$\bar{P} = qd\,\hat{z}$$

We need to represent Vp. by $\bar{P}$:

dot product $\bar{P}\cdot\hat{r} = qd\,\hat{z}\cdot\hat{r}$

$= qd\cos\theta$

note :

any unit vector . any unit vector = Cos(الزاوية المحصورة)

$$\therefore V_p = \frac{q d \cos\theta}{4\pi\epsilon r^2} = \frac{\bar{p} \cdot \hat{r}}{4\pi\epsilon r^2}$$

$$\therefore \boxed{V_p = \frac{\bar{p} \cdot \hat{r}}{4\pi\epsilon r^2}}$$

Mansoura University
Faculty of Engineering
Communication Engineering Department

Second Year Communication
Electromagnetic Fields
Sheet (1)

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[1] Given the electric field **E** = $2x\,a_x - 4y\,a_y$ (V/m), find the work done in moving a point charge +2C on the paths (Fig. 1):

(a) Path 1: from (2,0,0) to (0,0,0) and then to (0,2,0).

(b) Path 2: from (2,0,0) to (0,2,0) along the straight path joining the two points.

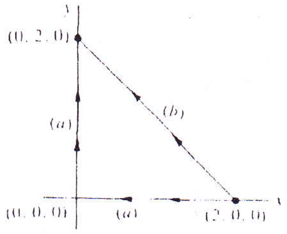

Fig. 1

[2] Find the potential at points A and B due to a point charge Q = 500 pC placed at the origin. Find the potential difference V_{AB}.

Hint: $r_A = 5$m and $r_B = 15$m

[3] For a line charge $\rho_l = (10^{-9}/2)$ C/m on the z-axis, find V_{AB}, where A is $(2,\pi/2,0)$ and B is $(4,\pi,5)$.

[4] A spherical region of space of radius R contains a charge Q that is distributed uniformly with constant volume charge density ρ C/m^3.

(a) Determine the stored electrostatic energy.

(b) Compare it to the energy of two point charges Q that are separated by a distance R.

[5] Two thin conducting half planes, at $\varphi = 0$ and $\varphi = \pi/6$, are insulated from each other along the z axis. Given the potential $V = (-60\,\varphi/\pi)$ V, find the stored energy between the half planes for $.1 \leq \rho \leq .6$ m and $0 \leq z \leq 1$ m.

[6] Two point charges +q and -q are placed at the points (0,0,d/2) and (0,0,-d/2), respectively. The charges form an electric dipole of moment **P** = $q\,d\,a_z$. Show that the potential at a far point (r,θ,φ) is $V = \mathbf{P}.a_r / (4\pi\varepsilon_0 r^2)$ for $r >> d$.

$qd\cos\theta$
$qd\,\hat{z}\cdot\hat{r}$